LITERATURE SYNTHESIS: ALGEBRA & SYMBOLISM

Algebra can be interpreted as “meaningless manipulations of meaningless symbols on a page” (Usiskin, 1997, p.346). This perception is upheld by many people, teachers and students who are not equipped with the tools and information which allow symbolism and algebra to fuse together. It is vital that students and their teachers make meaning of symbols, variables and unknowns in order to comprehend algebra and understand its importance in the mathematic curriculum. With emphasis on the insights, findings and misconceptions of symbols and equality in relation to primary mathematics pedagogy, this literature synthesis aims to provide a greater insight into the role and importance of symbolism in algebra.

The development of algebraic thinking in the early years relies heavily on a students’ understanding of the language associated with algebra (Ely & Adams, 2012; Uskiskin, 1997). The use of symbols, unknowns, placeholders and variables contribute to create the algebraic language which students often find confronting and somewhat difficult to understand; therefore “is an important source of difficulty in students’ transition from arithmetic to algebra” (Christou & Vosniadou, 2012, p. 2). In order to aid students in making this transition, teachers must firstly ensure students have a firm grasp on the concept and use of variables in mathematics (Hunter, 2012; Ely, et.al, 2012).

Some misconceptions associated with students’ understanding of variables include interpreting single letters in equations as being coded numbers; Christou and Vosniadou (2012) presented some common examples which represent this misunderstanding among primary students, these include upholding the idea that “values correspond to their positions in the alphabet” and variables “stand for abbreviated names of people or objects” (p.2). Another common misconception is “the notion that a single letter variable can only stand for a single number and variables represented by different letters cannot be the same number” (Hunter, 2012, p.256). This type of understanding portrays the idea that variables are fixed symbols which are representing a specific number In order to help students steer away from this narrow understanding, Ely and Adams (2012) suggest directing students to focus on the relationship between numbers and variables in a problem. This strategy is
grounded in the idea that it facilitates student’s algebraic reasoning so that they may recognise that a letter not only stands for an unknown value but a number of eligible values that are changeable (Ely & Adams, 2012). A practical implementation of this idea can be seen in the use of a “variable number line” whereby students “represent one variable quantity in terms of another” (Ely & Adams, 2012, p.36). Additionally, Yackel (1997) recommends using different concrete materials to engage students in reasoning about quantities, such as length and width, without having specific numerical values. This strategy helps students refrain from relying on their numerical thinking skills and encourages them to think more generally and relationally.

It is noted in the literature that in addition to variables, the ‘=’ symbol is significantly important for students to recognise and comprehend in order to develop algebraic thinking (e.g. Baratta, 2011; Morris, 2003; Warren, Mollison & Oestrich, 2009). It is described by Baratta (2011) as a threshold concept, meaning that without having grasped the concept “the learner could not progress” to develop algebraic reasoning (p.6). The equals sign “denotes a relationship of quantitative sameness between the two members of an equality sentence” but can often be mistaken by students as simply an operational tool which implies finding the answer to an equation (Morris, 2003, p.18). Although it is a symbol that is recognisable to most students from the beginning of formal education in mathematics, it is also a main contributor to a number of problems in relation to algebraic understanding in the later years of primary schooling.

Some common misinterpretations of the equals symbol include the idea that it is used in number sentences purely as a “command to perform...arithmetical operations,” (Morris, 2003, 18). Therefore when students are confronted with a number sentence which is not typical to a normal mathematic equation, students immediately perceive it as incorrect. Morris (2003) provides an example whereby a sentence such as “1+3=2+2” is generally perceived as inaccurate by most students who do not embody relational understanding of the equals symbol and what it represents.

In order to encourage students to move toward a relational understanding of the equals’ symbol, teachers must provide them with rich experiences where they can explore
the notion of equivalence. The Balance Model, founded by Filloy and Rajano (1989) (as cited in Baratta, 2011), is a tool used to represent equations and is “effective in giving meaning to abstract ideas” (Baratta, 2011) and further the understanding of equality in algebra (p.7). The balance model or strategy is noted by Warren, Mollison & Oestrich, 2009 to be “one of the most powerful strategies in solving the unknown in expressions” and is therefore extremely useful in promoting a conceptual understanding of the equals symbol (p.14). The balance strategy is also recognised by many other authors and researchers as a fundamental tool when teaching algebra (Baratta, 2011; Morris, 2003; Kamol & Har, 2010).

In conclusion, teachers should strive to avoid the onset and prevalence of common misconceptions when introducing the use of variables and symbolic representations in algebra. Much of the literature regarding this topic alerts teachers to provide opportunities for students to build upon their relational understanding and bridge the gap between arithmetical and algebraic thinking (e.g. Hunter, 2010; Nathan & Koedinger, 2000; Norton & Irvin, 2007). These opportunities should be available and catered to students at any age and ability with an emphasis on meaningful, contextually appropriate algebraic activities which aid their relational thinking in terms of symbolism in mathematics.

Words: 944
## LEARNING TRAJECTORY: ALGEBRA & SYMBOLISM

<table>
<thead>
<tr>
<th>TRAJECTORY LEVEL</th>
<th>CONCEPTUAL STRUCTURES</th>
<th>INSTRUCTIONAL TASKS</th>
</tr>
</thead>
</table>
| Lower: Aged 5-7  | Students are “developing comparative language... that assists in describing equivalent and non-equivalent situations” (Warren et.al, 2009, 11)  
Consider the equals sign as an operator rather than a relational symbol (Morris, 2003, p.18) | Comparing two different sets of concrete materials (blocks/liquid), encouraging students to use words such as “same as” & “equal to”  
Balance Model with concrete materials – using scales to establish knowledge of balancing both sides to become equal (Norton & Irvin, 2007, p.559). |
| Middle: Aged 8-10 | Students “use equivalent number sentences involving addition and subtraction to find unknown quantities” (AusVels, 2012)  
Students “use commutative and associative properties of addition and multiplication in mental computation” (VELS, 2011).  
Students “represent equations in a variety of different formats including equations with more than one number on the left hand side” (Morris, 2003, p.18) | Activities can involve concrete materials whereby students think of and figure out different ways to partition another number, finding its equivalent. E.g. There are 5 monkeys playing in two trees, how many different ways can the monkeys play in the trees? 3 monkeys in one tree and 2 in the other... 4 in one tree and 1 in the other etc.  
(for example, $3 + 4 = 4 + 3$ and $3 + 4 + 5$ can be done as $7 + 5$ or $3 + 9$)  
$6+3=4+2+3$  
Use technology based balancing models to represent how the equation above is plausible. |
<table>
<thead>
<tr>
<th>Upper: Aged 11-13</th>
<th>Students “use equivalent number sentences involving multiplication and division to find unknown quantities” (AusVels, 2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“Students establish equivalence relationships between mathematical expressions using properties such as the distributive property for multiplication over addition” (VELS, 2011).</td>
</tr>
<tr>
<td></td>
<td>Students are “Introduced to the concept of variables as a way of representing numbers using letters” (AusVels, 2012)</td>
</tr>
<tr>
<td></td>
<td>Students identify relationships between variables and describe them with language and words (for example, how hunger varies with time of the day) (VELS, 2011).</td>
</tr>
</tbody>
</table>

| 3 × 26 = 3 × (20 + 6) |

Using a variable number line to represent variables in terms of another e.g.

```
n-3 n-2 n-1 n  n+1 n+2 n+3
I I I I I I I I I I
```

| Students use “balance model thinking... to solve equations with variables on both sides” (Norton & Irvin, 2007, p.559). |
References


